Approximation Algorithms

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Knapsack Problem

Given n items of

integer weights: w_1 w_2 … w_n

values: p_1 p_2 … p_n

a knapsack of integer capacity W

find most valuable subset of the items that fit into the knapsack

Knapsack Problem by DP

W(i,j) is the minimum size knapsack required to make a profit of at least j using the first i items.

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$W(1,j) = w_1$ if $j \leq p_1$ else MAX

$W(i,j) = Min(W(i-1,j),\ w_i + W(i-1,j-p_i))$

Knapsack Problem by DP

$$W(i,j) = Min(W(i-1,j),\ w_i + W(i-1,j-p_i))$$

$O(n^2P)$

	1	2							nP
1									
2									
n									

Round Down

$K = P\varepsilon / n$ Note that $Kn = P\varepsilon$

p'_i = floor of p_i/K

$p'_i > p_i/K - 1$ and $Kp'_i \le p_i$

$KP'(O) > P(O) - Kn$

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S be the optimum solution to the rounded down version of the problem.

O be the optimum solution to the original problem.

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$P(S) \geq KP'(S) \geq KP'(O) \geq P(O)-Kn \geq OPT-P\varepsilon \geq OPT-\varepsilon OPT$

$$P(S) \geq (1-\varepsilon)OPT$$

Approximation Algorithms

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$P(S) \geq (1-\varepsilon)OPT$

$n/\varepsilon = P/K = P'$

$O(n^2P')$ which is $O(n^3/\varepsilon)$

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$P(S) \geq (1-\varepsilon)OPT$

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FPTAS - Fully Polynomial Time Approximation Scheme

Max Cut

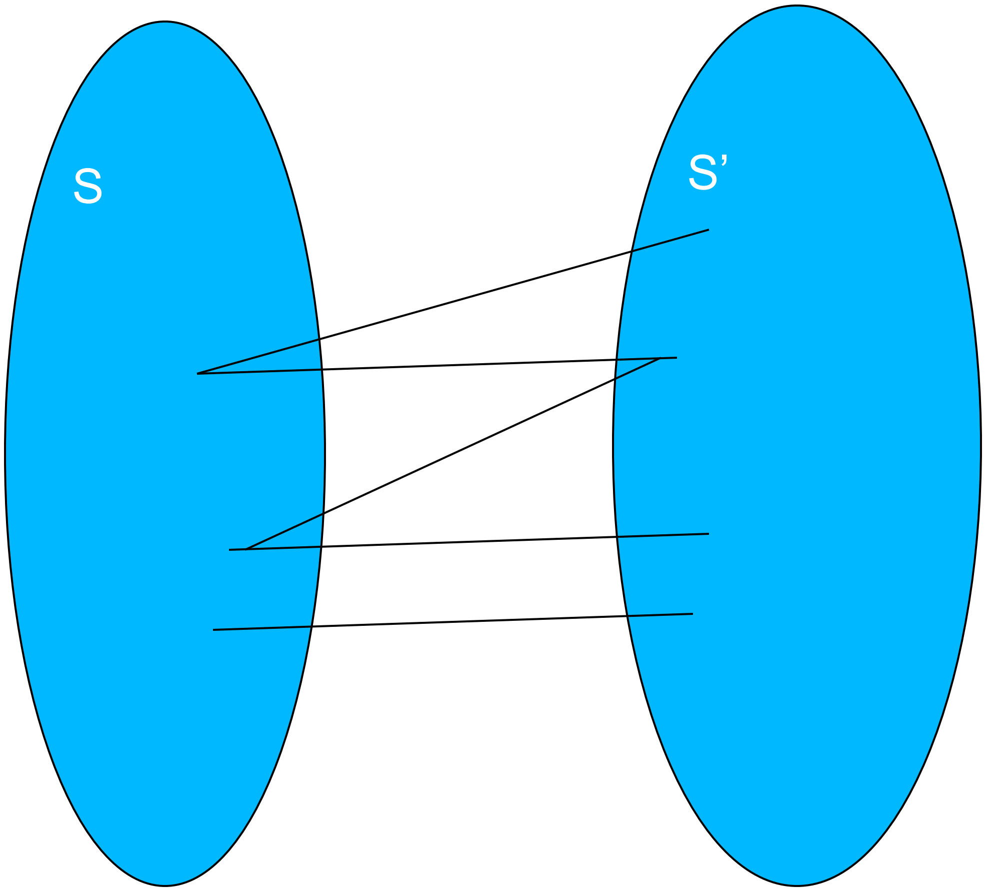
Given an weighted graph G=(V,E), a non empty proper subset of vertices S, is called cut.

We say that (S,S') is a cut , where S'=V\S. An edge is a cut edge , if exactly one end point is in S.

C(S)= The weight of the cut edges.

Cut in a Graph

Cut edges



Local Search

Start with any S

Try to move a node across the cut, if it in creases the weight of the cut, move it.

Stop, when you do not find any such node.

Analysis

u is in S , move it to S′ , the weight should not increase

$\sum_{v \varepsilon S'} w(u,v) \geq \sum_{v \varepsilon S} w(u,v)$

$2\sum_{v \varepsilon S'} w(u,v) \geq \sum_{v \varepsilon S} w(u,v) + \sum_{v \varepsilon S'} w(u,v)$

$2\sum_{v \varepsilon S'} w(u,v) \geq \sum w(u,v)$

$2.C(S) = \sum w(u,v) > OPT$

Max Cut

The best known algorithm achieves a bound of 0.878

This is the best we can do, if the unique game conjecture is true.

It has been proven to be NP-hard to approximate with an approximation ratio better than 16/17