Approximation Algorithms

Muralidhara V N
Associate Professor
IIIT Bangalore
murali@iiitb.ac.in

Introduction

- Efficient algorithms
- Polynomial time Algorithms
- NP- Hard Problems

Introduction

- Approximation Algorithms
- Parameterized Algorithms

- Online Algorithms
- Dynamic Algorithms

Approximation Algorithms

Consider minimization problems

A< α OPT

Vertex Cover

Given an undirected graph G=(V,E), a subset of vertices VC, is called vertex cover of given any edge there is at least one end point is in VC.

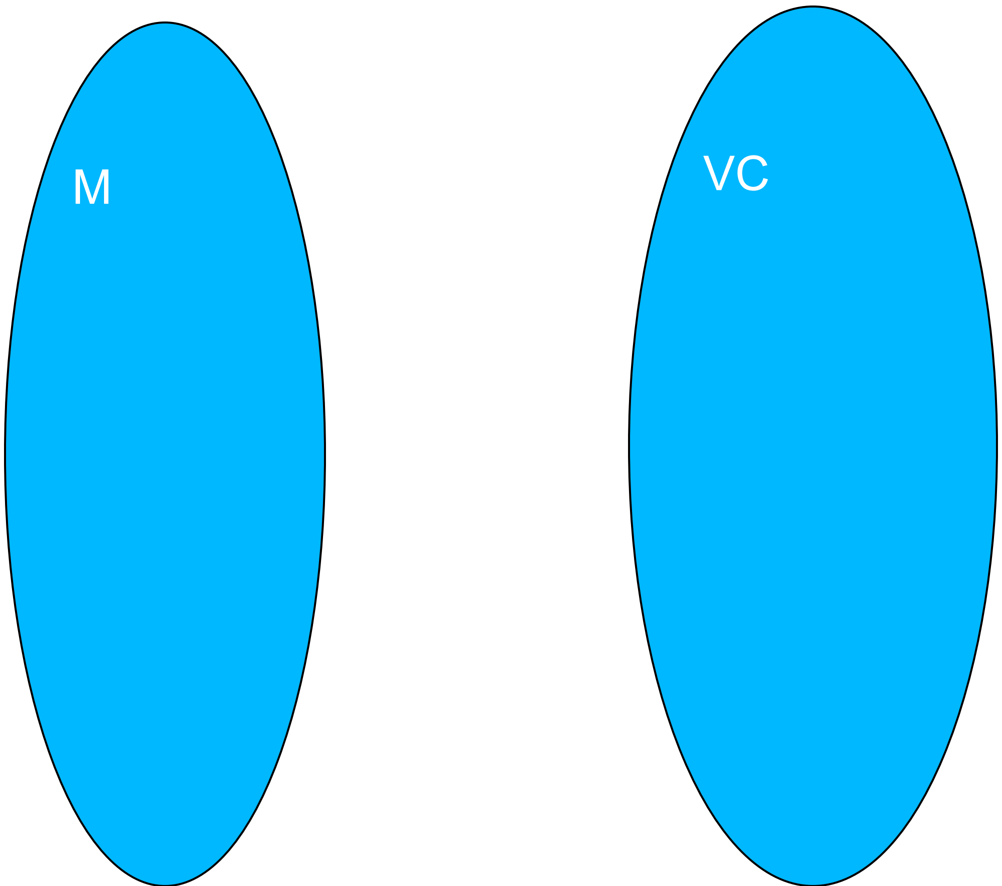
Problem to find the minimum vertex cover is NP-Hard, there is an easy 2 approximation algorithm.

Matching

Given an undirected graph G=(V,E), a subset of edges M, is called a matching if no two edges in M share a common end point.

Vertex Cover vs Matching

There is a one one mapping from any matching to any vertex cover.



M is lower bound to VC

$M \leq VC$

Find a maximal matching,

M is empty

Consider all edges in any order, if it can be added to M, add it.

Minimum VC algorithm

Algorithm A picks , both the end point of the maximal matching.

$A = 2\,M \leq 2\,VC$

We have a 2 approximation Algorithm.

Minimum VC algorithm

Can the analysis of the algorithm be improved to get a better bound ?

Minimum VC

Can we have α-approximation Algorithm with α< 2 ?

Minimum VC algorithm

Can we have α-approximation Algorithm with α< 2 ?

It cannot be approximated up to a factor smaller than 2 if the ***Unique games conjecture*** is true.

TSP

Given an weighted graph find a tour which visits every vertex once with minimum cost.

TSP

- **On a general graph is Hard to Approximate.**
- **There is a PTAS for Euclidean TSP.**

Metric TSP

Given an undirected weighted graph find a tour which visits every vertex once with minimum cost.

Weights of the graph is a metric.

$d(x,y) \leq d(x,z) + d(z,y)$

Metric TSP

Find MST

Double the edges of MST

All the nodes are of even degree.

Find the Euler cycle.

Metric TSP

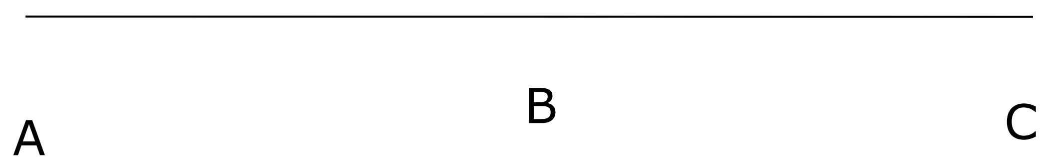
Find MST

Double the edges of MST

All the nodes are of even degree.

Find the Euler cycle.

Use short cut to remove duplicate nodes, you get TSP.



Metric TSP

Find MST

Double the edges of MST

All the nodes are of even degree.

Find the Euler cycle.

$A \leq 2MST < 2\ OPT$

Christofides-Serdyukov

Find MST

M be the minimum weight matching among all the nodes of odd degree.

Find the Euler cycle, in MST+M

$A \leq MST+M < OPT+1/2 OPT=1.5\ OPT$

Christofides-Serdyukov

Take OPT, remove all the even degree nodes, you get a cycle containing only odd degree vertices.

The alternate edges in this cycle is matching.

So $M < 1/2$ OPT.

A (Slightly) Improved Approximation Algorithm for Metric TSP

Anna R. Karlin, Nathan Klein, Shayan Oveis Gharan

Asymmetric TSP (2017)

A Constant-Factor Approximation Algorithm for the Asymmetric Traveling Salesman Problem

Ola Svensson, Jakub Tarnawski, László A. Végh